

# Charge Density Waves and Axion Strings from Weyl Semimetals

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We study dynamical instability and chiral symmetry breaking in three dimensional Weyl semimetals, which turns Weyl semimetals into “axion insulators”. Charge density waves (CDW) is found to be the natural consequence of the chiral symmetry breaking. The Goldstone mode of this charge density wave state is identified as the axion, which couples to electromagnetic field in the topological  $\theta \mathbf{E} \cdot \mathbf{B}$  form. Our main finding is that the “axion strings” can be realized as the (screw or edge) dislocations in the charge density wave, which provides a simple physical picture for the elusive axion strings. These axion strings carry gapless chiral modes, therefore they have important implications for dissipationless transport properties of Weyl semimetals with broken symmetry.

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**Introduction.** Topological insulators are among the most active research fields in condensed matter physics recently<sup>1-3</sup>. Among the remarkable aspects of topological insulators is the ubiquitous role played by Dirac fermions. In fact, most of the recently discovered topological insulators can be regarded as massive Dirac fermion systems with lattice regularization<sup>3</sup>. When the mass vanishes, we have massless Dirac fermions, which are two copies of Weyl fermion with opposite chiralities. The Dirac fermions, which obey the Dirac equation, are described by spinors with four components, while the Weyl fermions are two-component fermions described by the following simple Weyl equation

$$\pm \boldsymbol{\sigma} \cdot \mathbf{k} \psi = E \psi \quad (1)$$

where  $\pm$  is referred as chirality (+ for left handed, – for right handed). Since the Dirac fermion can be decomposed into two copies of Weyl fermion, the latter is more elementary building block in the real world and theoretical models. In fact, it is our current understanding of nature that the elementary fermions such as quarks and electrons fall into the Weyl fermion framework because the left-handed and right-handed fermions carry different gauge charges in the standard model of particle physics<sup>4</sup>.

It is worth noting that in writing down Eq.(1) we have assumed that the two “Weyl points”, at which the energy gap closes, are both located at  $\mathbf{k} = 0$ . In the particle physics context, this assumption seems to be natural, however, in condensed matter physics, without imposing symmetry constraints such as time reversal symmetry and inversion symmetry, Weyl points are generally located at different points in the momentum space. This fact has interesting consequences for Weyl fermion systems with broken symmetry, as we will show in this paper.

Although Weyl fermion plays a crucial role in the description of elementary fermions in nature, it has been studied in the condensed matter context only very recently<sup>5-21</sup>. Weyl semimetals in three dimensions (3d) are analogous to graphene<sup>22</sup> in two dimensions (2d) in the sense that both are described in terms of gapless fermions with approximately linear dispersion, but the 3d Weyl semimetals are richer in that they are more closely related to various fundamental phenomena such as the chiral anomaly<sup>10,11,21</sup>. Unlike the topological

insulators, whose transport is dominated by topologically protected surface states, in the Weyl semimetal the bulk transport is most important. Their unique semi-metallic behaviors in 3d can potentially be engineered for semiconductor industry.

Interaction effect plays a fundamental role in the dynamics of Weyl fermions. One possibility is the pairing interaction which leads to the superconducting instability. Qi, Hughes and Zhang’s fermi surface topological invariant<sup>23</sup> implies that if the pairing amplitudes have opposite sign for Weyl points with the opposite chirality, topological superconductors are obtained. Another consequences of interaction, which we will focus in this paper, is the spontaneous chiral symmetry breaking. Chiral symmetry breaking is the phenomenon of spontaneous generation of an effective mass of Weyl fermions, namely a pairing between the fermions (electrons) and anti-fermions (holes) with different chiralities. Due to chiral anomaly, the Goldstone mode  $\theta$  is coupled to electromagnetic field as  $\theta \mathbf{E} \cdot \mathbf{B}$ , therefore, this Goldstone mode is an “axion”<sup>24-27</sup>.

In this paper, we focus on the chiral symmetry breaking in Weyl semimetal and axion strings in condensed matter context, which have new features absent in particles physics. We would like to mention that axion string can be realized on the surface of topological insulators with a magnetic domain wall<sup>28</sup>, and axionic dynamics has been studied in topological magnetic insulators<sup>29,30</sup>. In the present paper we study a new route to axionic dynamics through chiral symmetry breaking induced by interaction effect. The resultant states are charge density wave (CDW) states, which are experimentally observable. One of our main findings is that the (screw or edge) dislocations of CDW are exactly the “axion strings”, which are important topological defects carrying gapless chiral modes. In these chiral modes electrons move solely in one direction without backscattering. In particles physics, axion strings have interesting cosmological implications such as the gravitational lenses effect<sup>31</sup>, but observable evidences of such axion strings are elusive. Axion strings in condensed matter systems have the advantage that they are much easier to detect. In the Weyl semimetal studied in the present paper, axion strings have important effects on the transport properties since they provide chiral modes supporting dissipationless transport.

*Dynamical chiral symmetry breaking.* We consider

the simplest model for dynamical chiral symmetry breaking, which nevertheless captures the most salient physical consequences. The Hamiltonian is given as  $H = H_0 + H_1$ , in which  $H_0$  is the free quadratic Hamiltonian given as  $H_0 = \sum_{\mathbf{k}\alpha\beta} c_{\mathbf{k}\alpha}^\dagger h_{\alpha\beta}(\mathbf{k}) c_{\mathbf{k}\beta}$ , and  $H_1 = -\frac{1}{2} \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} g(\mathbf{q})(c_{\mathbf{k}+\mathbf{q}\alpha}^\dagger c_{\mathbf{k}\alpha})(c_{\mathbf{k}'-\mathbf{q}\beta}^\dagger c_{\mathbf{k}'\beta}) = -\frac{1}{2} \sum_{\mathbf{q}} g(\mathbf{q}) \hat{\rho}_{\mathbf{q}} \hat{\rho}_{-\mathbf{q}}$ , where  $\alpha, \beta$  refer to all degrees of freedom other than the momentum  $\mathbf{k}$ ,  $\hat{\rho}_{\mathbf{q}} = \sum_{\mathbf{k}} c_{\mathbf{k}+\mathbf{q}}^\dagger c_{\mathbf{k}}$ , and  $-g(\mathbf{q})$  is the electron-electron interaction potential in the momentum space.

Let us first turn off the interaction term  $H_1$ . Suppose that the free Hamiltonian  $H_0$  has two Weyl points located at  $\mathbf{Q}_1$  and  $\mathbf{Q}_2$ . We shall not be concerned with the full form of Hamiltonian  $H_0$  since we mainly focus on the low energy physics, therefore we can expand

$$h(k) = \pm v_F \boldsymbol{\sigma} \cdot (\mathbf{k} - \mathbf{Q}_i) \quad (2)$$

where the  $\pm$  sign correspond to  $\mathbf{Q}_i$  with  $i = 1, 2$  respectively,  $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$  are three Pauli matrices. Several model Hamiltonians for  $H_0$  can be found, for example, in Ref.<sup>6,11</sup>.

For simplicity, in this paper we consider the case  $g(\mathbf{Q}) > 0$ , where  $\mathbf{Q} = \mathbf{Q}_1 - \mathbf{Q}_2$ . Taking the mean field value of  $H_1$ , we can easily see that a chiral condensation  $v = \sum_{\mathbf{k}} \langle c_{\mathbf{k}}^\dagger c_{\mathbf{k}+\mathbf{Q}} \rangle = \sum_{\mathbf{k}, \alpha} \langle c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}\alpha} \rangle$  leads to an energy gain  $-\frac{1}{2} g(\mathbf{Q}) |v|^2 < 0$ . This form of chiral condensation is a pairing of electrons and holes with total momentum  $\mathbf{Q}$ , which is reminiscent of the Larkin-Ovchinnikov-Fulde-Ferrell (LOFF) state of pairing between electrons with nonzero total momentum. More precisely, we can perform a Hubbard-Stratonovich transformation as

$$\begin{aligned} S &= \int d\tau [V \frac{|m|^2}{g(\mathbf{Q})} - \sum_{\mathbf{k}} (m c_{\mathbf{k}+\mathbf{Q}}^\dagger c_{\mathbf{k}} + m^* c_{\mathbf{k}}^\dagger c_{\mathbf{k}+\mathbf{Q}})] \\ &\approx \int d\tau [V \frac{|m|^2}{g(\mathbf{Q})} - \sum_{|\mathbf{p}| < \Lambda} (m c_{L,\mathbf{p}}^\dagger c_{R,\mathbf{p}} + m^* c_{R,\mathbf{p}}^\dagger c_{L,\mathbf{p}})] \quad (3) \end{aligned}$$

where  $c_{L,\mathbf{p}} = c_{\mathbf{Q}_1+\mathbf{p}}$  and  $c_{R,\mathbf{p}} = c_{\mathbf{Q}_2+\mathbf{p}}$ ,  $V$  is the volume of the system, and  $\Lambda$  is a momentum cutoff below which the linear Weyl fermion approximation is valid. Since we consider low energy dynamical symmetry breaking with  $m \ll \Lambda$ , contribution of fermions with  $|\mathbf{p}| > \Lambda$  to the effective action of  $m$  is negligible. Such limit we take, which does not qualitatively affect the physical conclusions, will simplify our calculation significantly. Integrating out fermions, we obtain the effective action of  $m$  at the mean field level as

$$S_{MF} = V\beta \frac{|m|^2}{g(\mathbf{Q})} - \text{Tr} \ln \det M \quad (4)$$

where  $V$  is the system volume,  $\beta = 1/\text{Temperature}$  is the period along the imaginary time, which will be taken to be very large since we are concerned with the zero temperature limit. The fermion matrix  $M$  is given by

$$M = \begin{pmatrix} \partial_\tau - v_F \boldsymbol{\sigma} \cdot \mathbf{p} & m \\ m^* & \partial_\tau + v_F \boldsymbol{\sigma} \cdot \mathbf{p} \end{pmatrix} \quad (5)$$

It follows that the saddle point equation in the zero temperature limit is

$$\frac{1}{2g} = \int \frac{d\omega d^3p}{(2\pi)^4} \frac{1}{\omega^2 + v_F^2 p^2 + |m|^2} \quad (6)$$

where  $g$  is the abbreviation of  $g(\mathbf{Q})$ . The solution to Eq.(6) is given as

$$\frac{1}{g_c} - \frac{1}{g} = \frac{1}{8\pi^2 v_F^3} |m|^2 \ln \frac{\Lambda^2 + |m|^2}{|m|^2} \quad (7)$$

where

$$g_c = \frac{8\pi^2 v_F^3}{\Lambda^2} \quad (8)$$

Because we are concerned with the cases with  $|m| \ll \Lambda$ , Eq.(7) is approximated by

$$\frac{1}{g_c} - \frac{1}{g} = \frac{1}{8\pi^2 v_F^3} |m|^2 \ln \frac{\Lambda^2}{|m|^2} \quad (9)$$

It can be seen from Eq.(9) that the coupling constant must exceed a critical value  $g_c$  to give a physical meaningful solution for  $|m|$ . Therefore, the interaction should be sufficiently strong to lead to dynamical symmetry breaking (or called ‘‘exciton condensation’’).

*Axion dynamics and topological theta term.* We have seen in the previous section that when  $g > g_c$ , chiral symmetry is spontaneously broken. The Ginzburg-Landau effective action of  $m$  can be expressed as

$$S_m = \int dt d^3x \left( \frac{1}{2} \gamma \partial_\mu m^* \partial^\mu m + \delta |m|^2 + \eta |m|^4 \right) \quad (10)$$

Let us write  $m = |m| \exp(i\theta)$ . In the symmetry breaking phase,  $\delta < 0$  and  $|m|$  develops a nonzero expectation value. From the Ginzburg-Landau effective action we can see that the fluctuations of  $|m|$  are gapped, while the  $\theta$  fluctuation is a gapless Goldstone mode. The Ginzburg-Landau action  $S_m$  is invariant under the transformation  $\theta \rightarrow \theta + \Delta\theta$ , but in fact this symmetry is broken by chiral anomaly, which endows a topological theta term to the effective action, as we explain as follows. We can perform a chiral transformation  $\psi_L \rightarrow e^{i\theta/2} \psi_L$ ,  $\psi_R \rightarrow e^{-i\theta/2} \psi_R$ , then the transformation of the dynamically generated mass is readily obtained as  $m \rightarrow m e^{-i\theta} = |m|$ . Although the action of low energy fermions is invariant under such chiral transformation, the integral measure is not<sup>32</sup>, thus the chiral transformation generates an anomalous term as<sup>32</sup>

$$\begin{aligned} S_{\text{anomaly}} &= \frac{e^2}{32\pi^2} \int dt d^3x \theta \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho} \\ &= \frac{e^2}{4\pi^2} \int dt d^3x \theta \mathbf{E} \cdot \mathbf{B} \quad (11) \end{aligned}$$

where we have used the unit with  $\hbar = c = 1$ .

Taking the above chiral anomaly into account, the gapless fluctuations of  $\theta$  is described by the following simple axionic effective action

$$S_\theta = \frac{f_a^2}{2} \int dt d^3x (\partial_\mu \theta)^2 + \frac{e^2}{4\pi^2} \int dt d^3x \theta \mathbf{E} \cdot \mathbf{B} \quad (12)$$

where the notation ‘‘ $f_a$ ’’ is deliberately chosen because it is analogous to the pion decay constant  $f_\pi$ , namely that  $f_a$  is the

“axion decay constant”. We can also define a normalized field  $a = f_a \theta$  and put Eq.(12) into another form

$$S_a = \frac{1}{2} \int dt d^3x (\partial_\mu a)^2 + \frac{e^2}{4\pi^2} \int dt d^3x \frac{a}{f_a} \mathbf{E} \cdot \mathbf{B} \quad (13)$$

The analogous effective action of the second term for pion is responsible for the famous two-photon decay of neutral pion. From the Ginzburg-Landau action in Eq.(10) we can see that  $f_a \sim |m|$ , therefore, we have the counterintuitive conclusion that when the chiral condensation goes to zero, the axion-photon coupling becomes strong.

Various topological responses can be calculated from the effective action given in Eq.(12). Taking derivative with respect to  $A_\mu$ , we have the current

$$j^\mu = \frac{e^2}{8\pi^2} \epsilon^{\mu\nu\lambda\rho} \partial_\nu \theta F_{\lambda\rho} \quad (14)$$

Such kind of responses have been studied in topological insulators<sup>28</sup>, and the case of Weyl semimetal is analogous in this respect. Let us consider the special case of a constant magnetic field  $B_z \mathbf{z}$  along the  $\mathbf{z}$  direction, then the charge density given by Eq.(14) is

$$j^0 = \frac{e^2}{4\pi^2} \partial_z \theta B_z \quad (15)$$

This is consistent with the following physical picture. In a constant magnetic field  $B_z \mathbf{z}$ , the motion of electrons in the  $xy$  plane is quantized into Landau levels with density of state  $eB_z/2\pi$ , then the problem is reduced to one dimensional, but we know that the charge density in one dimension can be found using the Goldstone-Wilczek formula<sup>33</sup>  $j^0_{1d} = \frac{1}{2\pi} \epsilon^{\mu\nu} \partial_\nu \theta$ , therefore, the three dimensional charge density is obtained as  $j^0 = (eB_z/2\pi)(\partial_z \theta/2\pi) = \frac{e^2}{4\pi^2} \partial_z \theta B_z$ .

*Phase of charge density wave is the axion.* Since we are concerned with low energy physics which is dominated by the two Weyl points, following the momentum space cutoff used in the previous sections, we expand the fermion operator as  $c_{\mathbf{r}} = e^{i\mathbf{Q}_1 \cdot \mathbf{r}} c_{L,\mathbf{r}} + e^{i\mathbf{Q}_2 \cdot \mathbf{r}} c_{R,\mathbf{r}} + \dots$ , where  $c_{R/L}$  are cut off in the momentum space at  $\Lambda$ , i.e.  $c_{L/R} = \sum_{|\mathbf{p}| < \Lambda} e^{i\mathbf{p} \cdot \mathbf{r}} c_{L/R,\mathbf{p}} + \dots$  and the “...” terms are high energy modes with  $|\mathbf{p}| > \Lambda$ . The charge density is given by

$$\rho(\mathbf{r}) = c_{\mathbf{r}}^\dagger c_{\mathbf{r}} = c_{L,\mathbf{r}}^\dagger c_{L,\mathbf{r}} + c_{R,\mathbf{r}}^\dagger c_{R,\mathbf{r}} + (c_{L,\mathbf{r}}^\dagger c_{R,\mathbf{r}} e^{-i\mathbf{Q} \cdot \mathbf{r}} + c_{R,\mathbf{r}}^\dagger c_{L,\mathbf{r}} e^{i\mathbf{Q} \cdot \mathbf{r}}) \quad (16)$$

The mean field expectation value of chiral condensation is  $v = \langle c_{L,\mathbf{r}}^\dagger c_{R,\mathbf{r}} \rangle = V^{-1} \sum_{\mathbf{p}} \langle c_{L,\mathbf{p}}^\dagger c_{R,\mathbf{p}} \rangle = m/g$ , which can be obtained from Eq.(3), therefore we have the charge density wave

$$\begin{aligned} \langle \rho(\mathbf{r}) \rangle &= \rho_0 + v \cos(\mathbf{Q} \cdot \mathbf{r} - \theta) \\ &= \rho_0 + \frac{|m|}{g} \cos(\mathbf{Q} \cdot \mathbf{r} - \theta) \end{aligned} \quad (17)$$

where  $\rho_0$  is the constant background, while the cosine term indicates that the chiral symmetry breaking induces a charge density wave state with wave vector  $\mathbf{Q}$ .

When the CDW wavelength  $2\pi/|\mathbf{Q}|$  is commensurate with the lattice constant, the chiral symmetry breaking transitions

can be thought of as generalized Peierls transitions, which induces dimerization, trimerization, etc. Unlike the infinitesimal Peierls instability in one dimension, the generalized Peierls transitions in three dimensional Weyl fermion systems can occur only when the coupling is stronger than a critical value [see Eq.(8)]. The vanishing density of state at the Weyl point is the reason for the difference.

The natural question is how to experimentally detect the CDW. It can be detected using bulk measurement, but it can also be detected by simpler surface measurements. The surfaces naturally inherit CDW from the bulk, therefore the CDW can be detected using scanning tunneling microscope (STM). Denoting the angle between the surface normal and  $\mathbf{Q}$  as  $\alpha$ , we can obtain that the surface CDW wavelength as  $\lambda_{2d} = \frac{2\pi}{|\mathbf{Q}| \sin \alpha}$ . When  $\alpha \rightarrow 0$ ,  $\lambda_{2d} \rightarrow \infty$ , i.e. in the cases with  $\mathbf{Q}$  perpendicular to the measured surface ( $\alpha = 0$ ), there is no surface CDW.

It is worth noting that the interaction effect and CDW was studied in 2d Dirac systems in Ref.<sup>34–36</sup>. More recently, interaction effect on the surface of weak topological insulators has been studied in Ref.<sup>37</sup>, in which CDW has also important physical consequences. The relation to chiral anomaly is absent in these studies because the systems considered there are in 2d, where the concept of chirality is lacking.

*Dislocations in charge density wave are axion strings.* Let us turn to the central part of this paper, namely the identification of CDW dislocations as axion strings, which may provide dissipationless chiral transport channel in 3d bulk materials. An axion string  $l$  is a one dimensional dislocation of axion field, around which the axion field  $\theta$  changes by  $2\pi$ , namely that

$$\int_C d\theta = 2\pi \quad (18)$$

where  $C$  is a small contour enclosing  $l$  clockwise,  $\theta$  is the axion field coupled to the gauge fields in the form of  $\theta \mathbf{E} \cdot \mathbf{B}$ . Axion strings are closely related to chiral anomaly, as was studied long ago in the work by Callan and Harvey<sup>38</sup> in the particle physics context. In the Weyl semimetals studied in the present paper, the axion strings have clear geometrical picture because  $\theta$  is exactly the phase of CDW. More explicitly, suppose that  $\mathbf{Q} = (Q_x, Q_y, Q_z) = (0, 0, Q)$ , then the maxima of charge density (peaks of CDW) is located at 2d planes ( $x, y, z_n$ ) with

$$z_n = \frac{\theta + 2\pi n}{Q} \quad (19)$$

where  $n$  = integer. When  $\theta$  is shifted, the location of maximal density plane follows the shifting of  $\theta$ . In fact, the shifting of a maximal density plane around the small loop  $C$  enclosing the axion string is readily obtained from Eq.(19) as

$$\int_C dz_n = \frac{\int_C d\theta}{Q} = \frac{2\pi}{Q} \quad (20)$$

which is exactly the wavelength of the CDW. The Burgers vector of the axion string as a dislocation of CDW is exactly  $(0, 0, 2\pi/Q)$ . For a general CDW wave vector  $\mathbf{Q}$ , the Burgers vector is readily obtained as  $2\pi\mathbf{Q}/|\mathbf{Q}|^2$ .

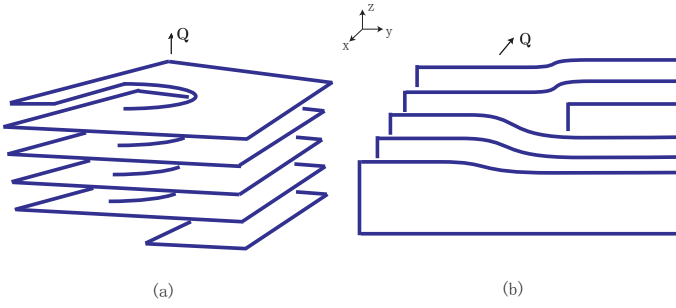


FIG. 1: Axion strings as dislocations of charge density wave. (a)Screw dislocation. (b)Edge dislocation. The delineated sheets are the peaks of CDW. In both (a) and (b), the axion string is along the  $z$  direction. The Burgers vector is parallel to the axion string in (a), while perpendicular to it in (b).

Let us refer the orientation of the axion string  $l$  as  $\hat{l}$ . According to the relative orientation of  $\hat{l}$  and  $\mathbf{Q}$ , the axion string appears as different types of dislocation. When  $\hat{l}$  is parallel with  $\mathbf{Q}$ , we have screw dislocation [Fig.(a)], while when  $\mathbf{Q}$  is parallel with  $\hat{l}$ , we have edge dislocation [Fig.(b)].

There are chiral modes propagating along the axion strings<sup>38</sup>, therefore, axion strings may serve as unique transport channels in a 3d materials with axionic dynamics. Such chiral modes carry dissipationless current just like the quantum Hall edge and quantum anomalous Hall edge states, but the former are more robust because they are buried in 3d bulk materials rather than placed at 2d sheets.

It is worth mentioning that dislocations in topological materials has also been studied in Ref.<sup>39</sup>, but we would like to emphasize several prominent differences between the axion strings studied here and the dislocation lines in weak topological insulators studied in Ref.<sup>39</sup>. First, in Ref.<sup>39</sup> the dislocations carrying gapless modes are indeed dislocations of crystal lattice, while in our paper the crystal lattice remains intact, and axion strings are just dislocations of CDW. Secondly, the gapless modes studied in Ref.<sup>39</sup> are helical modes, which are unstable towards back scattering if time reversal symmetry is broken, while the gapless modes living on the axion strings studied in the present paper are robust chiral modes. It is also worth noting that in Ref.<sup>11</sup> line dislocation with chiral modes was studied, but CDW is absent there, more importantly, the bulk is also gapless there and the coupling between dislocation mode and bulk mode can induce dissipation.

To conclude this section we remark that the formation of axion strings in Weyl semimetals can be triggered by rapidly lowering the temperature from  $T > T_c$  to  $T < T_c$ , where  $T_c$  is the critical temperature of chiral condensation (Kibble-Zurek

mechanism).

*Discussions and Conclusions.* It is worth mentioning that an attractive interaction  $-g(\mathbf{Q}) < 0$  at momentum  $\mathbf{Q}$  is needed for the chiral symmetry breaking. The values of  $g(\mathbf{Q})$  for various materials depend on the material details, which is not within the scope of this paper. It is useful to mention that an effective attractive electron-electron interaction can appear at some special momenta commensurate with the reciprocal lattice. Such electron-lattice coupling effect is responsible for the Peierls transitions in one dimensional systems, therefore we see that a sufficiently strong electron-lattice coupling is able to cause dynamical chiral symmetry breaking if  $\mathbf{Q}$  is commensurate with the reciprocal lattice.

Our model provides a geometrical picture of axion, which manifests itself as CDW. CDW phases have (edge and screw) dislocations, which are exactly the axion strings. The CDW and associated dislocations are much more visible than the elusive axion and axion strings in previous settings. One of our main results in this paper is the identification of axion strings as CDW dislocations, which has no analog in particle physics. The axion strings have one dimensional robust chiral modes along them, which have great potential applications if the chiral symmetry breaking (exciton condensation) of Weyl fermion is realized in experiment. In this paper we studied the general cases with  $\mathbf{Q} \neq 0$ . When  $\mathbf{Q} = 0$  there is no CDW associated with the chiral symmetry breaking, but the axion strings do exist and have important implication for 3d transport properties.

To conclude this paper, let us mention that the dynamical chiral symmetry breaking in Weyl semimetals discussed in this paper can also be realized in cold atom experiment. Although the anomalous  $\theta \mathbf{E} \cdot \mathbf{B}$  coupling is absent because atoms are neutral, the atom density wave and its dislocations can be realized. Because the flexible tunability in cold atom experiments, it is interesting to realize Weyl fermions and chiral symmetry breaking in cold atom systems.

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*Note added.* After this work was finished, we became aware of a related work<sup>40</sup> on symmetry breaking in Weyl semimetal by Zyuzin and Burkov, though CDW and axion strings were not studied. Due to nonzero density of states considered in their work, the gap equation is different from ours and an infinitesimal interaction will induce chiral condensation in that case.

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